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mercury itself can then be dried with bibulous paper, and after being gently heated becomes pure and bright.

20. I will only add, that the modifications which these experiments suggest in the construction of the common portable artificial horizon are very obvious. With regard to the ingenious arrangements which, after the completion of these experiments, Colonel Sabine informed me had been adopted some years ago by M. Schumacher for the artificial horizon, I found the form of the vessel, viz. the segment of a sphere, peculiarly objectionable, on account of the depth and the quantity of mercury employed: moreover, there is no possibility of easily cleaning the surface, or of removing the obnoxious film of amalgam so often alluded to.

On the Determination of the Orbit which a Satellite describes around its Planet. By M. Sawitsch, Professor of Astronomy in the University of St. Petersburg.

When there exist a great number of observations of a satellite, we may employ various combinations for finding with facility the elements of the orbit which the revolving body describes around its primary. It appears, however, that no method has yet been assigned for determining these elements from the smallest possible number of observations. The general theory which we owe to the illustrious Gauss, relative to the calculation of planetary orbits, undoubtedly applies also to the problem under consideration. This theory first leads to an approximate solution; after which it puts us in the way of obtaining such a degree of accuracy as the excellence of the observations demands. The provisional results, however, necessarily imply that the intervals between the times of observation are small, relatively to the complete revolution of the celestial body in its orbit. Such a supposition cannot be admitted when the question refers to a satellite, for in that case a single day may form a considerable part of the entire period of revolution. It is proposed, according to the following method, to determine the elements from three observations, the interval between the first and third being supposed not to exceed a few days.

Let t, t', t'', denote the times of the first, second, and third observations; Δ, Δ', Δ'', the observed distances of the satellite from the planet; and Q, Q′, Q″, the angles of position referred to the circle of the geocentric latitude of the planet. Moreover, let l, l', l'', b, b', b'', denote the geocentric longitudes and latitudes of the planet: and R, R′, R″, its corresponding linear distances from the earth.

All these quantities are supposed to be given either by observation or calculation; but the following are to be determined, viz.:—

$R(1+\delta)$, $R'(1+\delta')$, $R''(1+\delta'')$, the linear distances of the satellite from the earth, corresponding to the times t, t', t''.

τ, the period of the revolution of the satellite.

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ϱ, the angle under which the radius of the orbit of the satellite would appear at the mean distance of the planet from the sun.

ι, the inclination of the orbit of the satellite to a plane passing through the centre of the planet parallel to the ecliptic.

ϖ, the longitude of the ascending node of the orbit measured upon the same plane.

ϵ, the longitude of the epoch.

Let the motion of the satellite, as well as that of its primary, be referred to a system of rectangular co-ordinates, the origin of which is supposed to be at the centre of the earth. Let us suppose that the axes of X and Y are in the plane of the ecliptic; also let the positive axis of X be directed to the longitude l', the positive axis of Y to the longitude $90° + l'$, and the positive axis of Z to the north pole of the ecliptic.

Let X, Y, Z, X′, Y′, Z′, X″, Y″, Z″, be the co-ordinates of the planet, and $X + x$, &c., $X' + x'$, &c., $Z'' + z''$, &c., those of the satellite corresponding to the times t, t', t''. Let $d\,l$, $d\,l'$, $d\,l''$, represent the differences of geocentric longitude of the planet and satellite, and $d\,b$, $d\,b'$, $d\,b''$, the differences of latitude. Hence we have

$$\begin{aligned} X &= R \cos b \cos (l - l'), & X + x &= R\,(1 + \delta) \cos (b + d\,b) \cos (l - l' + d\,l) \\ Y &= R \cos b \sin (l - l'), & Y + y &= R\,(1 + \delta) \cos (b + d\,b) \sin (l - l' + d\,l) \\ Z &= R \,.\, \sin b, & Z + z &= R\,(1 + \delta) \sin (b + d\,b). \end{aligned}$$

Similar expressions may be obtained for the other co-ordinates of the planet and satellite.

We have also manifestly

$$\begin{aligned} d\,l &= \frac{\Delta \,.\, \sin Q}{\cos b}; & d\,l' &= \frac{\Delta' \,.\, \sin Q'}{\cos b'}; & d\,l'' &= \frac{\Delta'' \,.\, \sin Q''}{\cos b''}; \\ d\,b &= \Delta \cos Q; & d\,b' &= \Delta' \cos Q'; & d\,b'' &= \Delta'' \cos Q''. \end{aligned}$$

Let us now transport the system of co-ordinates parallel to itself from the centre of the earth to the centre of the planet. The co-ordinates of the satellite relative to the centre of the planet will then be represented by $x, y, z, x', y', z', x'', y'', z''$. Remarking that $R\delta$, $R'\delta'$, $R''\delta''$, $\sin d\,l$, $\sin d\,b$, $\sin d\,l'$, $\sin d\,b'$, $\sin d\,l''$, $\sin d\,b''$ may be considered as small quantities of the first order, the squares and products of which may be neglected; and bearing in mind further that $l - l'$, $l'' - l'$, do not amount, during an interval of several days, to more than a few minutes of space, we have without sensible error,

$$\left.\begin{aligned} x &= R \,.\, \delta \cos b - R \,.\, \sin b \,.\, \cos Q \,.\, \Delta \,.\, \sin 1'' \\ y &= R \,.\, \sin Q \,.\, \Delta \,.\, \sin 1'' \\ z &= R \,.\, \delta \sin b + R \cos b \,.\, \cos Q \,.\, \Delta \,.\, \sin 1'' \end{aligned}\right\} \text{(A)}$$

Similar expressions may be obtained for $x', y', z', x'', y'', z''$.

Let n denote the area of the triangle formed by the radii vectores of the satellite corresponding to the second and third observations, with the cord which unites their extremities. Let n', n'' represent the areas of the similar triangles corresponding to the first and third, and to the second and first observations. Since the satellite revolves in a plane passing through the centre of the planet, which coincides with the origin of the co-ordinates x, y, z, the following equations are deducible:—

$$n x - n' x' + n'' x'' = 0 \qquad (1)$$

$$n y - n' y' + n'' y'' = 0 \qquad (2)$$

$$n z - n' z' + n'' z'' = 0 \qquad (3)$$

Substituting in (2) the values of y, y', y'', as already determined, we have

$$\frac{n'}{n} = \frac{\text{R} \,.\, \Delta \,.\, \sin \text{Q}}{\text{R}' \,.\, \Delta' \,.\, \sin \text{Q}'} + \frac{n''}{n'} \; \frac{\text{R}'' \,.\, \Delta'' \,.\, \sin \text{Q}''}{\text{R}' \,.\, \Delta' \,.\, \sin \text{Q}'} \qquad (\alpha)$$

Multiplying (1) by sin b', and (3) by cos b', and subtracting the one result from the other, bearing in mind at the same time, that as the geocentric latitudes of the principal planets are generally small, their alterations, $b - b'$, $b'' - b'$, in the space of a few days are inconsiderable, we have

$$\frac{n'}{n} = \frac{\text{R} \,.\, \Delta \,.\, \cos \text{Q}}{\text{R}' \,.\, \Delta' \,.\, \cos \text{Q}'} + \frac{n''}{n} \,.\, \frac{\text{R}'' \,.\, \Delta'' \,.\, \cos \text{Q}''}{\text{R}' \,.\, \Delta' \,.\, \cos \text{Q}'} \qquad (\beta)$$

If we suppose the satellite to revolve in a circular orbit whose radius is r, we may employ the equations (α) and (β) in finding τ the period of revolution.

Let v'', v', v, be the angles described by the radius of the satellite during the intervals of time $t' - t$, $t'' - t$, $t'' - t'$. We have then

$$n = \frac{1}{2} r^2 \sin v, \; n' = \frac{1}{2} r^2 \sin v', \; n'' = \frac{1}{2} r^2 \sin v''$$

$$\frac{n'}{n} = \frac{\sin v'}{\sin v}, \quad \frac{n''}{n} = \frac{\sin v''}{\sin v}$$

$$v = \frac{360^\circ}{\tau} (t'' - t'), \; v' = \frac{360^\circ}{\tau} (t'' - t), \; v = \frac{360^\circ}{\tau} (t' - t)$$

It is clear, then, that if an approximate value of τ was known, we might easily find v, v', v'', and hence also the relations $\frac{n'}{n}$, $\frac{n''}{n}$. Now it is always possible to obtain such a value of τ by successive trials. Assuming $\tau = 3$ or 6 or 10, &c. days, we calculate the corresponding values of $\frac{\sin v'}{\sin v}$, $\frac{\sin v''}{\sin v}$, and then we shall see which of the hypotheses satisfies best the equality of $\frac{\sin v'}{\sin v}$ and $\frac{n'}{n}$ as determined by (α) or (β). The corresponding value of τ will express the approximate value of the period of revolution. In order to avoid useless calculation, it will be desirable to take into consideration

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the three angles of position Q, Q′, Q″, which will serve to show if the interval of time $t'' - t$ between the first and third observations approaches nearly or not to the time of a complete revolution. Knowing τ approximately, and repeating the calculation for two hypotheses of that element, differing slightly from each other, we may obtain a more accurate result, and in this manner we may approach nearer and nearer to the true value. When the angles Q, Q′, Q″, are very small, or very nearly equal to 180°, it will be advantageous to employ equation (β) instead of equation (α).

The following example, based upon three of M. Otto Struve's observations of *Neptune's* satellite, will serve to illustrate the practical utility of the method:—

Date.	M. T. Pulkowa.	Angular Distances.	Angle of Position with respect to the Circle of Latitude.
1847. Sept. 25	$10^h\ 21^m = t$	$15''{\cdot}92 = \Delta$	$228^\circ\ 45' = Q$
28	$12^h\ 9^m = t'$	$17''{\cdot}30 = \Delta'$	$57^\circ\ 42' = Q'$
30	$9^h\ 36^m = t''$	$5''{\cdot}07 = \Delta''$	$164^\circ\ 51' = Q''$

Hence

$$t' - t = 3^d{\cdot}075,\quad t'' - t' = 1^d{\cdot}894,\quad t'' - t = 4^d{\cdot}969.$$

Moreover, the ephemeris of *Neptune* gives

$$\log R = 1{\cdot}46450,\ \log R' = 1{\cdot}46505,\ \log R'' = 1{\cdot}46540$$

It will be seen at once that the time of a complete revolution exceeds five days, for in 4·969 days the angle of position (Q″) has not nearly resumed its original value (Q). We may then commence by supposing $\tau = 5{\cdot}5$ and $\tau = 6$. The former of these hypotheses gives for $\frac{\sin v'}{\sin v}$ a numerical value by far *too small* in comparison with its equivalent $\frac{n'}{n}$ calculated from the formula (α). Taking $\tau = 6$, we obtain for $\frac{\sin v'}{\sin v}$ a value *too great.* If we now assume $\tau = 5{\cdot}7$ and $\tau = 5{\cdot}8$ we obtain

$$\log\frac{n'}{n} - \log\frac{\sin v'}{\sin v} = +0{\cdot}00643\ (\tau = 5{\cdot}7)$$

$$\log\frac{n'}{n} - \log\frac{\sin v'}{\sin v} = -0{\cdot}02390\ (\tau = 5{\cdot}8)$$

Thus the one hypothesis is 0·00643 in *excess*, and the other, 0·02390 in *defect*, and hence by proportional parts we find $\tau = 5{\cdot}72$. Repeating the calculation with $\tau = 5{\cdot}72$, we shall find that this hypothesis is erroneous only to the extent of $+\ 0{\cdot}00113$, and we obtain definitively $\tau = 5{\cdot}724$ days. By a discussion of the totality of eighteen observations made in the years 1847–49, M. A. Struve found $\tau = 5{\cdot}876$ days, a result which accords sufficiently well with that deduced by an approximative process from three isolated observations.

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When once we know the value of τ we may easily determine the radius of the orbit of the satellite.

Let r denote the linear value of this radius; A the semi-major axis of the orbit of the planet around which the satellite revolves; v'' the angle included between the two radii whose extremities are determined by the co-ordinates x, y, z, x', y', z'. We have then

$$\cos v'' = \frac{x\,x' + y\,y' + z\,z'}{r^2}$$

substituting for x, y, z, x', y', z' their values (A); bearing in mind also that $\varrho = \frac{r}{\mathrm{A} \sin 1''}$ and that $r^2 = x^2 + y^2 + z^2 = x'^2 + y'^2 + z'^2$, we finally obtain

$$\varrho = \sqrt{\tfrac{1}{2}\,\mathrm{K}^2 \pm \sqrt{\tfrac{1}{4}\,\mathrm{K}^4 - \mathrm{M}^2}}$$

in which

$$\mathrm{M}^2 = \frac{\left(\frac{\mathrm{R}}{\mathrm{A}} \cdot \Delta\right)^2 \left(\frac{\mathrm{R}'}{\mathrm{A}} \cdot \Delta'\right) . \sin^2 (\mathrm{Q}' - \mathrm{Q})}{\sin^2 v''}$$

$$\mathrm{R}^2 = \frac{\left(\frac{\mathrm{R}}{\mathrm{A}} \cdot \Delta\right)^2 + \left(\frac{\mathrm{R}'}{\mathrm{A}} \cdot \Delta'\right)^2 - 2 \left(\frac{\mathrm{R}}{\mathrm{A}} \cdot \Delta\right) \left(\frac{\mathrm{R}'}{\mathrm{A}} \cdot \Delta'\right) \cos (\mathrm{Q}' - \mathrm{Q}) \cos v''}{\sin^2 v''}$$

It is easy to select the proper sign of the quantity under the radical in the expression for ϱ, since the latter quantity is essentially greater than $\frac{\mathrm{R} \,.\, \Delta}{\mathrm{A}}$, $\frac{\mathrm{R}' \,.\, \Delta'}{\mathrm{A}}$, or $\frac{\mathrm{R}'' \,.\, \Delta''}{\mathrm{A}}$. We may determine ϱ in a similar way by combining the third observation with the first or second. The most favourable combination is that in which $\sin v$ has the greatest value.

In the above example we have,

$$\mathrm{Q} = 228^\circ\,45' \qquad \mathrm{Q}' = 57^\circ\,42'$$

$$v'' = 193^\circ\,22'; \qquad \frac{\mathrm{R}}{\mathrm{A}} \cdot \Delta = 15''\cdot445; \qquad \frac{\mathrm{R}'}{\mathrm{A}} \cdot \Delta' = 16''\cdot806$$

$$\tfrac{1}{2}\,\mathrm{K}^2 = 206\cdot3; \qquad \mathrm{M}^2 = 30624; \qquad \sqrt{\tfrac{1}{4}\,\mathrm{K}^2 - \mathrm{M}^2} = 109\cdot2$$

$$\varrho^2 = 315\cdot5; \qquad \varrho = 17''\cdot762$$

M. A. Struve has deduced $\varrho = 17''\cdot969$ from the totality of 18 observations.

We have now to calculate the planetocentric longitudes and latitudes of the satellite, λ, λ', λ'', θ, θ', θ'', with respect to a plane passing through the centre of the planet parallel to the ecliptic. The expressions of the three planetocentric co-ordinates x, y, z, in which the direction of the axis of x is determined by the longitude l, lead to the following equations:—

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$$\varrho \cdot \cos\theta \cdot \cos(\lambda - l) = \xi \cos b - \sin b \,\frac{R}{A} \cdot \Delta \cdot \cos Q \qquad (a)$$

$$\varrho \cdot \cos\theta \cdot \sin(\lambda - l) = \frac{R}{A} \cdot \Delta \cdot \sin Q \qquad (b)$$

$$\varrho \sin\theta = \xi \sin b + \cos b \cdot \frac{R}{A} \cdot \Delta \cdot \cos Q \qquad (c)$$

where

$$\xi = \pm\sqrt{\varrho^2 - \left(\frac{R}{A} \cdot \Delta\right)^2}$$

Dividing (*b*) by (*a*) we find an expression for $\tan(\lambda - l)$, and as l or the geocentric longitude of the planet for the time t is known, we thus obtain the value of λ. The equation (*c*) serves to calculate θ, the planetocentric latitude of the satellite.

If we suppose the direction of the axis of x to be determined by the longitude l', and then by the longitude l'', we shall obtain equations analogous to (*a*), (*b*), (*c*), which will serve to calculate λ', θ', and λ'', θ'', by means of the data relative to the second and third observations.

When we have determined $\lambda, \theta, \lambda', \theta'$, the following formulæ will serve to calculate the inclination i, and the longitude ☊ of the ascending node of the orbit of the satellite upon a plane passing through the centre of the planet parallel to the ecliptic.

$$\sin . \left(\frac{\lambda' + \lambda}{2} - \text{☊}\right), \tan i = \tfrac{1}{2}(\tan\theta' + \tan\theta) \sec \tfrac{1}{2}(\lambda' - \lambda)$$

$$\cos . \left(\frac{\lambda' + \lambda}{2} - \text{☊}\right), \tan i = \tfrac{1}{2}(\tan\theta' - \tan\theta) \operatorname{cosec} \tfrac{1}{2}(\lambda' - \lambda)$$

We might obtain i and ☊ in a similar manner by combining λ'', θ'', with λ', θ', or with λ, θ.

In our example,

by combining λ', θ', with λ, θ, we have ☊ $= 264° \; 46'$; $i = 37° \; 35'$

— — λ'', θ'' — λ', θ' — ☊ $= 296 \; 36$; $i = 30 \; 28$

The difference of these two determinations, although sufficiently great, does not depend upon calculation. It arises from very small errors to which the best observations are liable, and which may exercise a very sensible influence according to the positions of the satellite with respect to the nodes at the times of observation. We may remark that the combination of the values $\lambda, \theta, \lambda', \theta'$, is not favourable for determining ☊, since the satellite was in those positions nearly $-90°$ and $+90°$ from the node. The combination of the second observation with the third is the most favourable for this purpose. As regards i both combinations are almost equally good. We admit, then,

$$\text{☊} = 296°, \; i = 34°$$

With these results, the equations

$$\tan u = \tan . (\lambda - \text{☊}) \sec i ; \quad \Lambda = u + \text{☊}$$

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will give the arguments of latitude u, u', u'', and the planetocentric longitudes Λ, Λ', Λ'', of the satellite in its own orbit.

In our example, $\Lambda = 201^\circ\ 34'$, $\Lambda' = 34^\circ\ 0'$, $\Lambda'' = 148^\circ\ 16'$. Designating by μ the mean daily motion of the satellite, we have—

$$\mu = \frac{\Lambda'' - \Lambda}{t'' - t} = 61^\circ{\cdot}72 \text{ by the interval of nearly 5 days.}$$

$$\mu = \frac{\Lambda' - \Lambda}{t' - t} = 62{\cdot}64 \quad — \quad — \quad 3 \ —$$

$$\mu = \frac{\Lambda'' - \Lambda'}{t'' - t'} = 60{\cdot}35 \quad — \quad — \quad 2 \ —$$

Whence the probable value of the mean daily motion,

$$\mu = 61^\circ{\cdot}72,$$

and the period of revolution,

$$\tau = \frac{360^\circ}{\mu} = 5{\cdot}832 \text{ days} = 5^d\ 19^h\ 10^m.$$

With $\mu = 61^\circ{\cdot}72$ we find the planetocentric longitude of the satellite upon its orbit, in 1847, September 28, at $12^h\ 9^m$ mean time to be—

By the first observation and the interval of	$3{\cdot}075^d$	$31^\circ\ 14'$
second — —	0	34 0
third — —	$1{\cdot}894$	31 26
	Mean value	32 13

The corresponding argument of latitude is $96^\circ\ 13'$. Subtracting the motion in 11·01 days, we have $136^\circ\ 45'$ for the argument of latitude on the 11th of September, at $11^h\ 55^m$ mean time. M. A. Struve determines the same argument, by means of eighteen observations, to be equal to $128^\circ\ 26'$. We are, therefore, in error only to the extent of $8^\circ\ 20'$.

As the inclination of the orbit of *Neptune* to the ecliptic is $= 1^\circ\ 47'$, and as the longitude of the ascending node of the orbit is $= 130^\circ\ 7'$, we hence deduce the longitude of the ascending node of the satellite upon the orbit of *Neptune* to be $= 296^\circ\ 37'$, and the inclination to be $= 35^\circ\ 44'$. We have now to explain the modes of verifying what has been already found, and to assign results of greater accuracy.

Since the trustworthiness of the preceding calculations depends upon the precise knowledge of τ the period of revolution, which has only been determined approximately in the first instance, it will be desirable to ascertain whether this value accords sufficiently well with the value $\tau = \frac{360^\circ}{\mu}$. The latter value ought to be in general the more accurate of the two, and if it differs considerably

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from τ as determined by means of (α) or (β), it will be necessary to repeat the calculations.

Assuming $\mu = 61^\circ{\cdot}72$, we have $\tau = 5{\cdot}832$ days. By the first calculations we obtained $\tau = 5{\cdot}724$. The difference does not amount to more than 0·108 days.

Let us now inquire into the degree of approximation of the angle ϱ as already determined. With $\mu = 61^\circ{\cdot}72$ we have—

$$v'' = 189^\circ\ 40'\ ;\quad Q' - Q = 188^\circ\ 58'\ ;\quad \frac{R\,\Delta}{A} = 15''{\cdot}445\ ;\quad \frac{R'\,\Delta'}{A} = 16''{\cdot}806\ ;$$

whence $\varrho = 20''{\cdot}40$

$$v = 116^\circ\ 54'\ ;\quad Q'' - Q' = 107^\circ\ 8'\ ;\quad \frac{R''\,\Delta''}{A} = 4{\cdot}929\ ;\quad \frac{R'\,\Delta'}{A} = 16''{\cdot}806\ ;$$

whence $\varrho = 18{\cdot}27$

$$v' = 306^\circ\ 42'\ ;\quad Q'' - Q = 296^\circ\ 54'\ ;\quad \frac{R''\,\Delta''}{A} = 4{\cdot}929\ ;\quad \frac{R\,\Delta}{A} = 15{\cdot}445\ ;$$

whence $\varrho = 18{\cdot}02$

As the errors Δ, Δ', Q, Q', produce upon the determination of ϱ an influence so much greater, as $\sin v''$ is less, we may take—

$$\frac{20''{\cdot}40 \sin v'' + 18''{\cdot}27 \sin v + 18''{\cdot}02 \sin v'}{\sin v'' + \sin v + \sin v'} = 18''{\cdot}35,$$

for the probable value of ϱ, as deducible from three observations. A new calculation of ☊ and i would diminish only in a small degree the uncertainty of these elements.

It remains for us to say a few words upon the mode of obtaining the elliptic elements of the satellite. Although four observations suffice *theoretically* for that purpose, still, considering the small value of the excentricity and the sensible influence of errors of observation, it will be impossible to attain the object by means of these data alone. Before undertaking the calculation of the elliptic elements it is desirable to ascertain the circular elements, and *that* to a certain degree of precision, so as to treat their corrections as small quantities of the same order of magnitude as the excentricities. It is necessary, besides, to have a long series of good observations, in order to deduce from them the elliptic elements of the orbit in a trustworthy manner.

The first thing to be done is to ascertain the degree of accuracy with which the observations are satisfied by the circular elements. For this purpose, it will be necessary to calculate with these elements the angle of position Q_0 and the apparent angular distance Δ_0 for the time of each observation. Representing by Δ, Q, the values of these quantities as deducible from the corresponding observations, we then form the differential quantities $\Delta - \Delta_0$, $Q - Q_0$, which will serve to exhibit the errors of the existing hypothesis respecting the orbit of the satellite. Now if the orbit be supposed to be in reality an ellipse, $\Delta - \Delta_0$ and $Q - Q_0$ may

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generally be expressed in terms of the corrections of the circular elements and the excentricity and longitude of the perihelion. Let e be the excentricity and π the longitude of the peri-planet; then, putting $e \cos \pi = \psi$ and $e \sin \pi = \eta$, we may express $\Delta - \Delta_0$ and $Q - Q_0$ in terms of seven unknown small quantities, viz.: the corrections of the five elements already involved in the circular hypothesis and the quantities ψ and η. An equation of condition involving these quantities may be formed for every observation, whether of distance or position; and from the totality of such equations, the values of the unknown quantities may be deduced by the application of the method of least squares.

Extracts of a Letter from M. Otto Struve to the Astronomer Royal, dated $\frac{15}{3}$ January, 1853.

"As to what regards Villarceau's remarks on the differences in right ascension of feeble comets, as observed by different instruments and methods, I am quite of opinion that similar differences must exist, and that for different reasons, but principally produced by the different optical power of the telescopes. The nature of these differences is, perhaps, not yet sufficiently explained for all different methods of observation; but, generally speaking, the most powerful telescopes must furnish the most exact results; and for this reason, I think, Villarceau has done wrong to attribute a higher precision to the Königsberg observations than to those made with the Northumberland telescope, especially as Wichmann himself thinks the accuracy of his observations of D'Arrest's comet has seriously been injured by the faintness of the object, while Challis does not mention at all that these observations have given him particular trouble."

"Last month the University of Dorpat celebrated the fiftieth anniversary of its renewed existence. On this occasion I wrote a dissertation, entitled *Narratio de Parallaxi Stellæ α Lyræ.* In this tract I have given, besides some general remarks on the determination of parallaxes, the result of fifteen months' observation on the parallax of *α Lyræ.* The comparison star used by me is identical with that of my father; but the result does not so well agree with the former, as I thought it would do. My observations, which, as to accuracy, appear to be considerably superior to those of my father, make the parallax by the

Angles of position $= +0''\cdot145 \pm 0''\cdot016$
By the distances $= +0''\cdot140 \pm 0''\cdot025$

"In combining these results with the former values obtained by W. Struve and Peters, the parallax of *α Lyræ* is found in the mean $= +0''\cdot1549 \pm 0''\cdot0120$.

"I have indicated in the same tract several reasons, which will account in part for the larger parallax found by my father, but the